ACOUSTICS

An Introduction to Its Physical Principles and Applications

Allan D. Pierce

School of Mechanical Engineering
Georgia Institute of Technology

McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
Johannesburg London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

CHAPTER ONE
THE WAVE THEORY OF SOUND

Acoustics is the science of sound, including its production, transmission, and effects.[†] (In present usage, the term *sound* implies not only the phenomena in air responsible for the sensation of hearing but also whatever else is governed by analogous physical principles. Thus, disturbances with frequencies too low (*infrasound*) or too high (*ultrasound*) to be heard by a normal person are also regarded as sound. One may speak of underwater sound, sound in solids, or structure-borne sound. Acoustics is distinguished from optics in that sound is a mechanical, rather than an electromagnetic, wave motion.

The broad scope of acoustics as an area of interest and endeavor can be ascribed to a variety of reasons. First, there is the ubiquitous nature of mechanical radiation, generated by natural causes and by human activity. Then, there is the existence of the sensation of hearing, of the human vocal ability, of communication via sound, along with the variety of psychological influences sound has on those who hear it. Such areas as speech, music, sound recording and reproduction, telephony, sound reinforcement, audiology, architectural acoustics, and noise control have strong association with the sensation of hearing. That sound is a means of transmitting information, irrespective of our natural ability to hear, is also a significant factor, especially in underwater acoustics. A variety of applications, in basic research and in technology, exploit the fact that the transmission of sound is affected by, and consequently gives information concerning, the medium through which it passes and intervening bodies and inhomogeneities. The physical effects of sound on substances and bodies with which it interacts present other areas of concern and of technical application.

Some indication of the scope of acoustics and of the disciplines with which it is associated can be found in Fig. 1-1. The first annular ring depicts the

[†] Definitions in the present text conform to ANSI/ASA S1.1, 2013 Edition, *American National Standard Acoustical Terminology* (Acoustical Society of America Standards Store, onlne site). Selected symbols for physical quantities conform to *American National Standard Letter Symbols and Abbreviations for Acoustics* (IEEE Xplore, 260.4–1996, online site).

traditional subdivisions of acoustics, and the outer ring names technical and artistic fields to which acoustics may be applied. (The chart is not intended to be complete, nor should any rigid interpretation be placed on the depicted proximity of any subdivision to a technical field. An extensive survey of the scope of acoustics can be found in (T. Rossing, editor) *Springer Handbook of Acoustics*(2nd Edition, Springer, 2014).

The present text, while intended as an introduction to acoustics, is concerned primarily with the physical principles underlying the discipline rather than with a summary of the current state of knowledge and technology in its many subfields. The general and specialized principles chosen for discussion are those which have found application in one or more of the following subfields: atmospheric acoustics, underwater acoustics, musical acoustics, ultrasonics, architectural acoustics, aeroacoustics, nonlinear acoustics, environmental acoustics, and noise control. For the most part, the selected subject matter is limited to sound in fluids, e.g., air and water.

We begin with a discussion of the wave theory of sound.

1-1 A LITTLE HISTORY

The speculation that sound is a wave phenomenon grew out of observations of water waves. The rudimentary notion of a *wave* is an oscillatory *disturbance* that moves away from some source and transports no discernable amount of matter over large distances of *propagation*. The possibility that sound exhibits analogous behavior was emphasized, for example, by the Greek philosopher Chrysippus (c. 240 B.C.), by the Roman architect and engineer Vetruvius (c. 25 B.C.), and by the Roman philosopher Boethius (A.D. 480–524). The wave interpretation was also consistent with Aristotle's (384–322 B.C.) statement[†] to the effect that air motion is generated by a source, "thrusting forward in like manner the adjoining air, so that the sound travels unaltered in quality as far as the disturbance of the air manages to reach."

A pertinent experimental result, inferred with reasonable conclusiveness by the early seventeenth century, with antecedents dating back to Pythagoras (c. 550 B.C.) and perhaps farther, is that the air motion generated by a vibrating body sounding a single musical note is also vibratory and of the same frequency as the body. The history of this is intertwined with the development of the laws for the natural frequencies of vibrating strings and of the

† M. R. Cohen and I. E. Drabkin, *A Source Book in Greek Science*, Harvard University Press, Cambridge, Mass., 1948, pp. 289, 293–294, 307–308. Aristotle's statements on acoustics are also reprinted by R. B. Lindsay (ed.), *Acoustics: Historical and Philosophical Development*, Dowden, Hutchinson, and Ross, Stroudsburg, Penn., 1972, pp. 22–24. For a detailed account of the early history of acoustics, see F. V. Hunt, *Origins of Acoustics*, Yale University Press, New Haven, Conn., 1978. Hunt, p. 26, states that the above-cited aristotelian statement was probably written by Straton of Lampsacus (c. 340–269 B.C.).

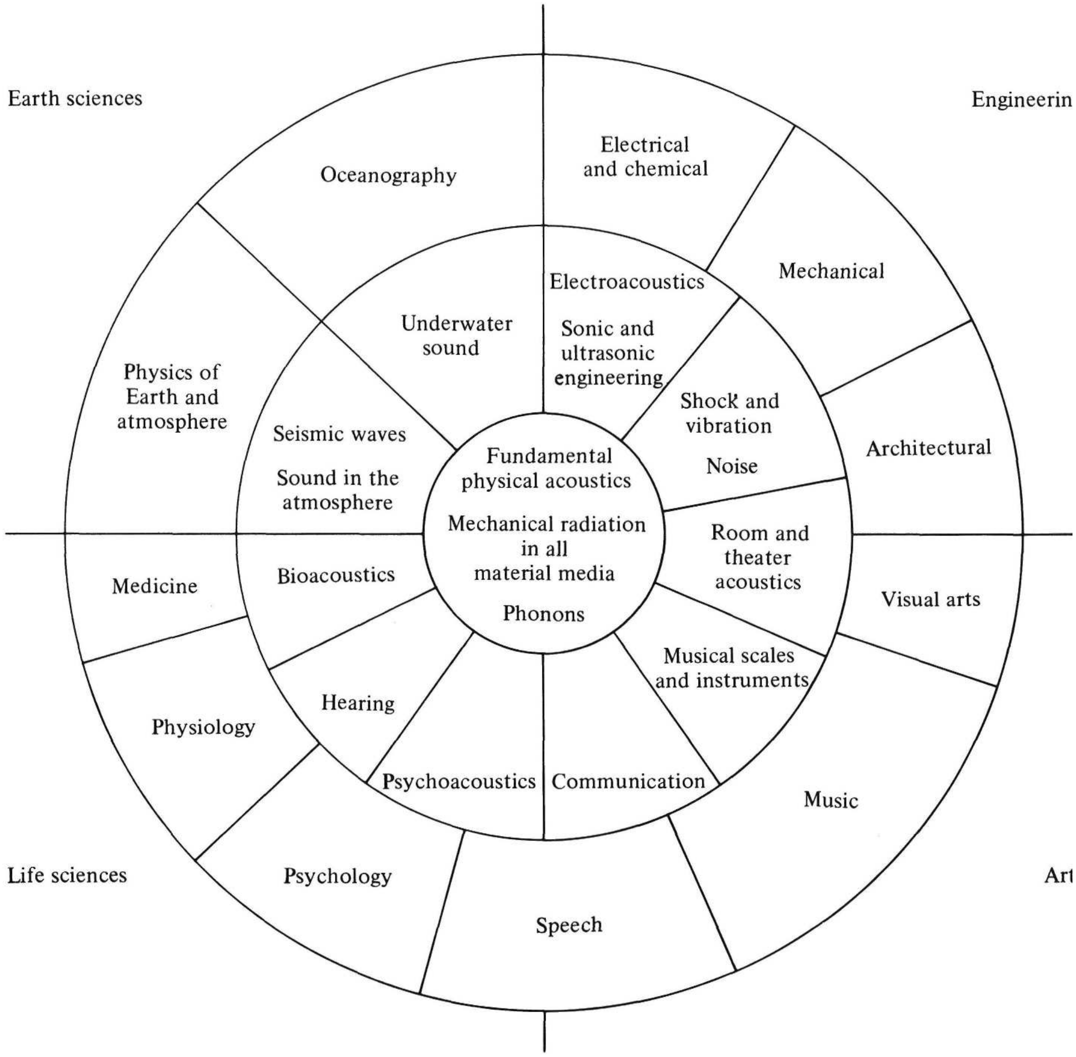

Figure 1-1 Circular chart illustrating the scope and ramifications of acoustics. [*Adapted from R. B. Lindsay, J. Acoust. Soc. Am.* **36**:*2242 (1964)*.]

physical interpretation of musical consonances.[‡] Principal roles were played by Marin Mersenne (1588–1648), a French natural philosopher often referred to as the "father of acoustics," and by Galileo Galilei (1564–1642), whose *Mathematical Discourses Concerning Two New Sciences* (1638) contained[§] the most lucid statement and discussion given up until then of the frequency equivalence.

Mersenne's description in his *Harmonie universelle* (1636) of the first absolute determination of the frequency of an audible tone (at 84 Hz) implies that he had already demonstrated that the absolute-frequency ratio of two

[‡] S. Dostrovsky, "Early vibration theory: physics and music in the Seventeenth Century," *Arch. Hist. Exact Sci.* **14**:169–218 (1975).

[§] The pertinent passages are reprinted in Lindsay, *Acoustics*, pp. 42–61, especially p. 48.

vibrating strings, radiating a musical note and its octave, is as 1:2. The perceived harmony (consonance) of two such notes would be explained if the ratio of the air oscillation frequencies is also 1:2, which in turn is consistent with the source-air-motion-frequency-equivalence hypothesis.

The analogy with water waves was strengthened by the belief that air motion associated with musical sounds is oscillatory and by the observation that sound travels with a finite speed. Another matter of common knowledge was that sound bends around corners, which suggested diffraction, a phenomenon often observed in water waves. Also, Robert Boyle's (1660) classic experiment[†] on the sound radiation by a ticking watch in a partially evacuated glass vessel provided evidence that air is necessary, either for the production or transmission of sound.

The wave viewpoint was not unanimous, however. Gassendi[‡] (a contemporary of Mersenne and Galileo), for example, argued that sound is due to a stream of "atoms" emitted by the sounding body; velocity of sound is speed of atoms; frequency is number emitted per unit time.

The apparent conflict[§] between ray and wave theories played a major role in the history of the sister science optics, but the theory of sound developed almost from its beginning as a wave theory. When ray concepts were used to explain acoustic phenomena, as was done, for example, by Reynolds and Rayleigh[‖] in the nineteenth century, they were regarded, either implicitly or explicitly, as mathematical approximations to a then well-developed wave theory; the successful incorporation of geometrical optics into a more comprehensive wave theory had demonstrated that viable approximate models of complicated wave phenomena could be expressed in terms of ray concepts. (This recognition has strongly influenced twentieth-century developments in architectural acoustics, underwater acoustics, and noise control.)

The mathematical theory of sound propagation began with Isaac Newton (1642–1727), whose *Principia*[¶] (1686) included a mechanical interpretation of

---

[†] R. Boyle, *New Experiments, Physico-Mechanical, Touching the Spring of the Air*, 2d ed., 1662, Experiment 27, reprinted by Lindsay, pp. 68–73. Lindsay gives a modern interpretation of Boyle's experiment in "Transmission of sound through air at low pressure," *Am. J. Phys.* **16**:371–377 (1948).

[‡] R. B. Lindsay, "Pierre Gassendi and the revival of atomism in the Renaissance," *Am. J. Phys.* **13**:235–242 (1945).

[§] A. E. Shapiro, "Kinematic optics: A study of the wave theory of light in the Seventeenth Century," *Arch. Hist. Exact Sci.* **11**:134–266 (1973).

[‖] O. Reynolds, "On the refraction of sound by the atmosphere," *Proc. R. Soc. Lond.* **22**: 531–548 (1874); J. W. Strutt, Baron Rayleigh, *The Theory of Sound*, vol. 2, 1878; 2d ed., 1896; reprinted by Dover, New York, 1945, secs. 286–290.

[¶] There are several editions and translations. One generally available is the revision by F. Cajori of Andrew Motte's translation (1729), from Latin into English, of the third edition (1726): *Newton's Principia: Motte's Translation Revised*, University of California Press, Berkeley, 1934, reprinted 1947. Lindsay reprints passages from an 1848 edition of Motte's translation. Dostrovsky, "Early vibration theory," gives a detailed deciphering of Newton's analysis. The first such was given by Euler (1744).

sound as being "pressure" pulses transmitted through neighboring fluid particles. Accompanying diagrams (see Fig. 1-2) illustrated the diverging of wave fronts after passage through a slit. The mathematical analysis was limited to waves of constant frequency, employed a number of circuitous devices and approximations, and suffered from an incomplete definition of terminology and concepts. It was universally acknowledged by his successors as difficult to decipher, but, once deciphered, it is recognizable as a development consistent with more modern treatments. Some textbook writers, perhaps for pedagogical reasons, stress that Newton's one quantitative result† that could then be compared with experiment, i.e., the speed of sound, was too low by about 16 percent. The reason for the discrepancy and how it was resolved is discussed below (Sec. 1-4), but it is a relatively minor aspect of the overall theory, whose resolution required concepts and experimental results that came much later.

Substantial progress toward the development of a viable theory of sound propagation resting on firmer mathematical and physical concepts was made during the eighteenth century† by Euler (1707–1783), Lagrange (1736–1813), and d'Alembert (1717–1783). During this era, continuum physics, or field theory, began to receive a definite mathematical structure. The wave equation emerged in a number of contexts, including the propagation of sound in air. The theory ultimately proposed for sound in the eighteenth century was incomplete from many standpoints, but the modern theories of today can be regarded for the most part as refinements of that developed by Euler and his contemporaries.

In Secs. 1-2 to 1-5 the basic equations for the simplest realistic model of sound propagation in fluids are described. Two of them, the conservation-of-mass equation and Euler's equation of motion for a fluid, come without alterations from the eighteenth century; the third, which relates pressure and density, is a nineteenth-century development. The model leads to the same wave equation as developed in the eighteenth century but gives a value for the sound speed that in most contexts of interest agrees satisfactorily with experiment. Although this model is approximate and gives no account of sound absorption, its predictions are often a good approximation to reality. Because of its simplicity, it is the one most often used unless there is some positive indication that the refinements contained in more complicated models are necessary for the problem at hand.

† Quotations from textbooks and a defense are given by H. Whiteside, "Newton's derivation of the velocity of sound," *Am. J. Phys.* **32**:384 (1964).

† A detailed commentary on the Euler era is given in a sequence of articles by C. A. Truesdell that appear as editor's introductions to volumes of *Leonhardi Euleri Opera Omnia*, ser. 2, Orell Füssli, Lausanne and Zurich, 1954, 1955, and 1960: "Rational fluid mechanics, 1687–1765," vol. 12, pp. IX–CXXV; "The theory of aerial sound, 1687–1788," vol. 13, pp. XIX–LXXII; "Rational fluid mechanics, 1765–1788," vol. 13, pp. LXXIII-CII; "The rational mechanics of flexible or elastic bodies, 1638–1788," vol. 11, pt. 2.

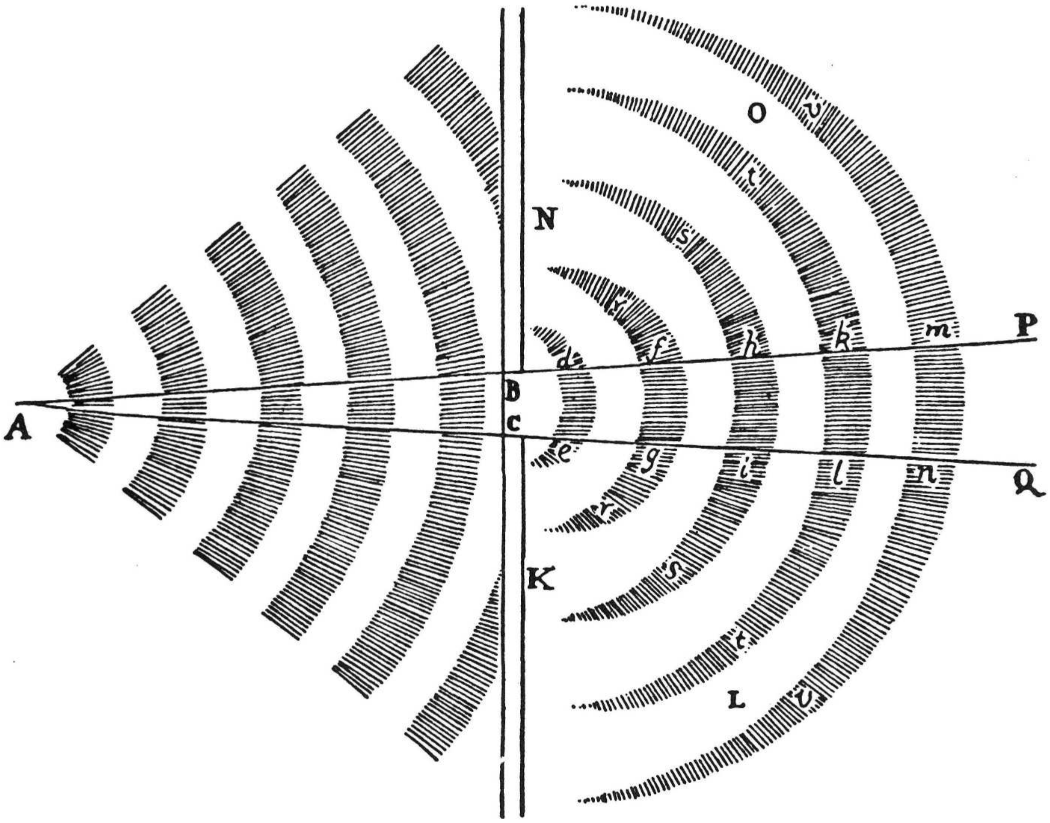

Figure 1-2 Sketch in Newton's *Principia* (1686) of the passage of waves through a hole. The source is at point A; the hole is described by points B and C; de, fg, hi, etc., describe the ''tops of several waves, divided from each other by as many intermediate valleys or hollows." (Adapted from *Sir Isaac Newton's Principia, 4th ed., 1726, reprinted 1871, by MacLehose, Glasgow, p. 359.)*

1-2 THE CONSERVATION OF MASS

For a fixed volume V (see Fig. 1-3a) inside a fluid (e.g., air or water), the net mass in V at any time t can be taken as the volume integral of a *density* $\rho(\boldsymbol{x}, t)$, representing a local average (or expected value) of mass per unit volume in the vicinity of a spatial[‡] point $\boldsymbol{x}$. Conservation of mass requires the time rate of change of this mass to equal the net mass per unit time entering (minus that leaving) the volume V through the confining surface S. The net mass per unit time leaving through a small area element ΔS with outward unit normal vector $\boldsymbol{n}(\boldsymbol{x}_S)$ and centered at point $\boldsymbol{x}_S$ on S is identified

[‡] The text uses the spatial (eulerian) description rather than the material (lagrangian) description (in which fluid dynamic variables are considered as functions of material or initial coordinates and time). Both descriptions originated with Euler; the terminology eulerian and lagrangian originated with Dirichlet (1860). (Truesdell, "Rational Fluid Mechanics, 1687–1765," p. cxx.)